

# Statistics

## Spring 2023

### Lecture 43



Feb 19-8:47 AM

The college randomly selected 240 students and 84 of them were on financial aid.  $n = 240$   
 $x = 84$

1) what proportion of these students are on financial aid?  
 $\hat{p} = \frac{x}{n} = \frac{84}{240} = .35$      $\hat{q} = 1 - \hat{p} = .65$

2) find 90% conf. interval for the proportion of all students that are on financial aid.  
 C-level: .9    1-Prop Z Int     $.299 < P < .401$   
 $30\% < P < 40\%$

3) find the margin of error.

$$E = \frac{.401 - .299}{2} = \frac{.102}{2} = .051 \approx 5\%$$

$$\hat{p} = \frac{.401 + .299}{2} = \frac{.7}{2} = .35 = 35\%$$

May 2-7:16 AM

Minimum Sample Size needed to construct  
Conf. interval for population proportion:

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

with  
Some  
Algebra

$$n = \hat{p}\hat{q} \left( \frac{Z_{\alpha/2}}{E} \right)^2$$

If decimal,  
Always round-up

If  $\hat{p}$  and  $\hat{q}$  are unknown, then we use .5  
for them, and

$$n = .25 \left( \frac{Z_{\alpha/2}}{E} \right)^2$$

May 2-7:27 AM

Suppose the college wish to be 95% confident  
and margin of error not to exceed 4%,  
what is the minimum sample size needed?

$$n = \hat{p}\hat{q} \left( \frac{Z_{\alpha/2}}{E} \right)^2$$

$$= (.35)(.65) \left( \frac{1.960}{.04} \right)^2 = 546.2275$$

$$n \approx 547$$



$$Z_{\alpha/2} = \text{invNorm}(.975, 0, 1) = 1.960$$

May 2-7:32 AM

60% of 352 college students randomly selected had a part-time job.  $n=352$   
 $\hat{p} = .6 \rightarrow \hat{q} = .4$

1) How many of these students had a part-time job?  
 $x = n\hat{p} \quad x = 352(.6) = 211.2$   
 if decimal, Round-up  $x = 212$

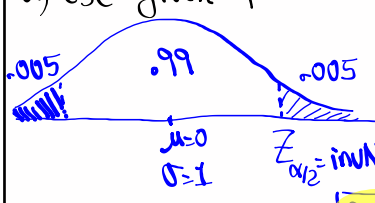
2) Find Conf. interval for the proportion of all college students that have part-time job.  
 NO C-level  $\Rightarrow$  use .95  $\rightarrow$  1-PropZInt  $.551 < P < .653$   
 $55\% < P < 65\%$

3) Find the margin of error.  
 $E = \frac{.653 - .551}{2} = .051 \approx 5\%$   
 $\hat{p} = \frac{.653 + .551}{2} = .602 \approx 60\%$

May 2-7:38 AM

4) Find minimum Sample Size needed if we wish to be 99% confident and margin of error not to exceed 4%.

a) use given  $\hat{p}$



$$n = \hat{p}\hat{q} \left( \frac{Z_{\alpha/2}}{E} \right)^2$$

$$= .6 \cdot .4 \left( \frac{2.576}{.04} \right)^2$$

$$= 995.3664$$

$$n \approx 996$$

$Z_{\alpha/2} = \text{invNorm}(.995, 0, 1) = 2.576$

b) Assume  $\hat{p}$  &  $\hat{q}$  are unknown.  
 $n = .25 \left( \frac{Z_{\alpha/2}}{E} \right)^2 = .25 \left( \frac{2.576}{.04} \right)^2$   
 $= 1036.84$   
 $n \approx 1037$

May 2-7:47 AM

In a survey of 185 college students, 28% of them were STEM majors.

$$\begin{aligned} n &= 185 \\ \hat{p} &= .28 \\ \hat{q} &= .72 \end{aligned}$$

1) Find 98% conf. interval for the prop. of all college students that are STEM majors.

C-level: .98      1-Prop Z Int       $.204 < p < .358$

$$x = n\hat{p} = 185(.28) = 51.8$$

$$x = 52$$

$$20\% < p < 36\%$$

2) Find the margin of error.

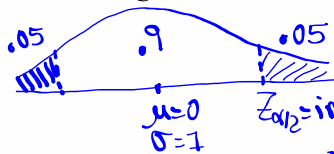
$$E = \frac{.358 - .204}{2} = .077 \approx 8\%$$

May 2-7:55 AM

4) Find minimum sample size needed if we wish to construct conf. interval for Pop. prop. with 90% c-level and error not to exceed 5%.

$$n = \hat{p}\hat{q} \left( \frac{Z_{\alpha/2}}{E} \right)^2$$

a) use given  $\hat{p}$  &  $\hat{q}$ .



$$= 1.645$$

$$= .28 \cdot .72 \left( \frac{1.645}{.05} \right)^2$$

$$n = 218.214$$

$$n \approx 219$$

b) Assume  $\hat{p}$  and  $\hat{q}$  are unknown.

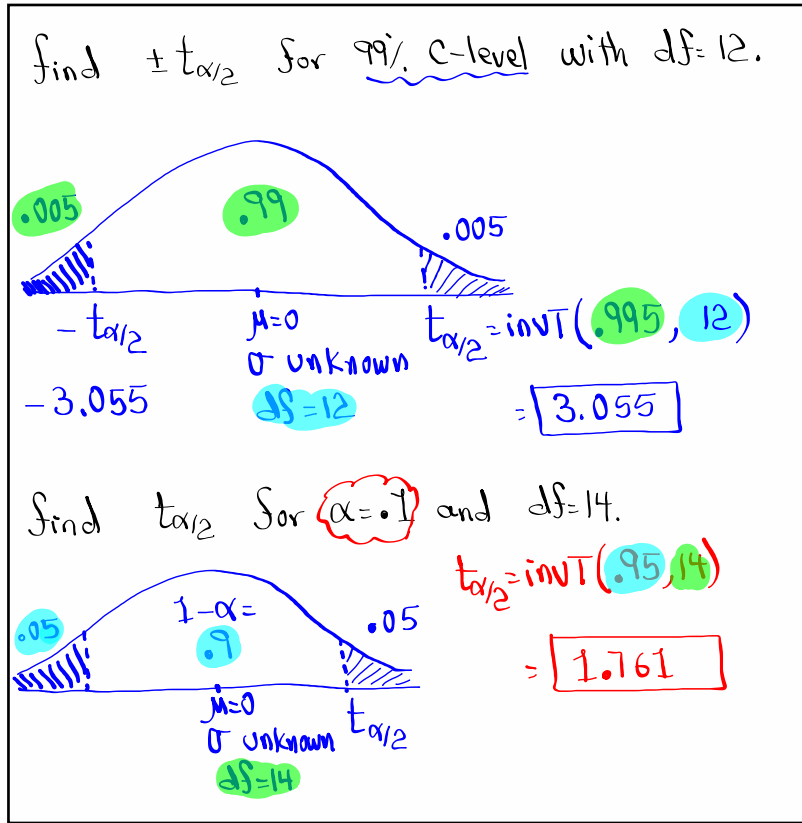
$$n = .25 \left( \frac{Z_{\alpha/2}}{E} \right)^2 = .25 \left( \frac{1.645}{.05} \right)^2$$

$$= 270.6025$$

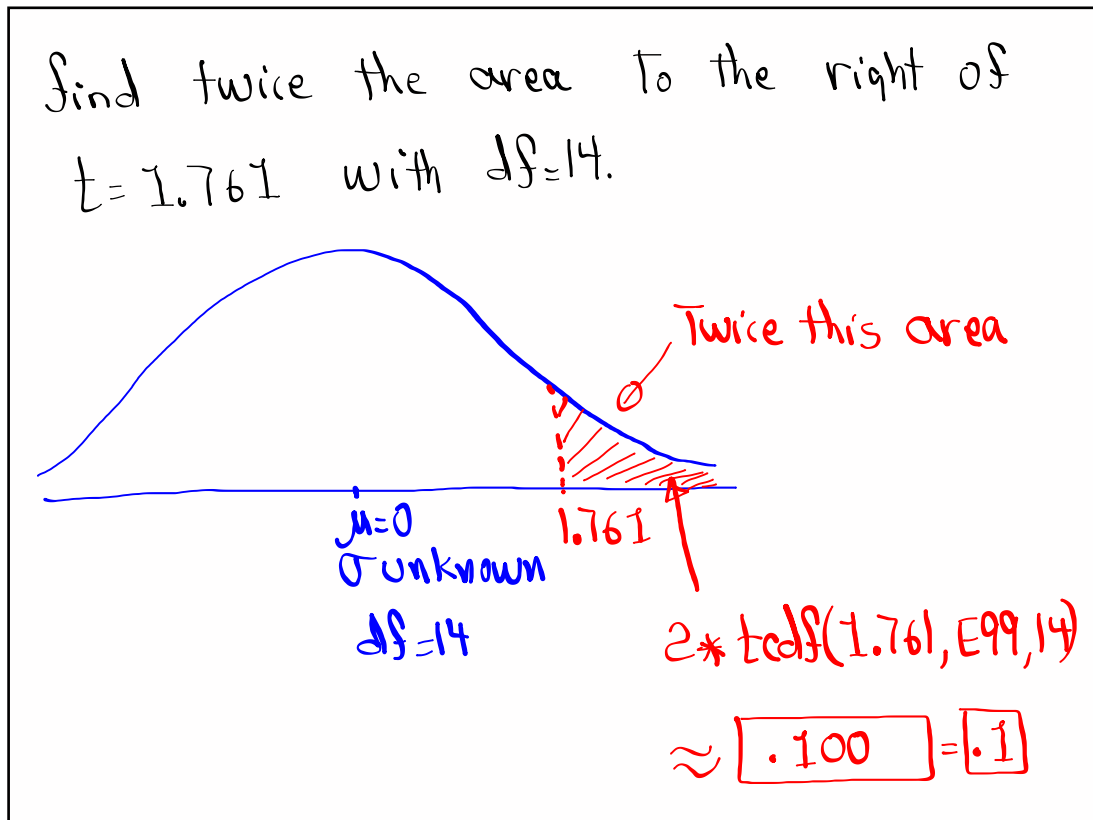
$$n = 271$$

May 2-8:03 AM





May 2-8:11 AM

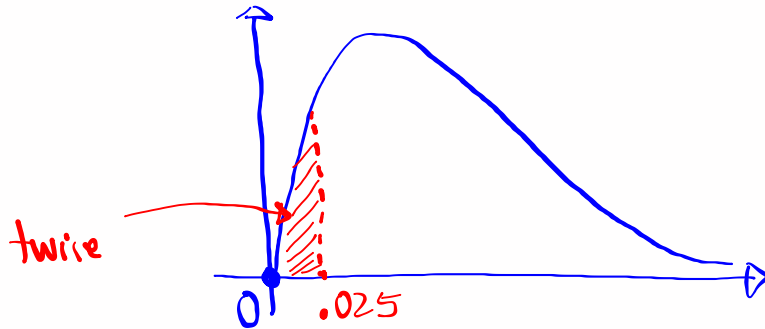


May 2-8:17 AM

Find twice the area to the left of

$\chi^2 = .025$  with  $df = 8$ .

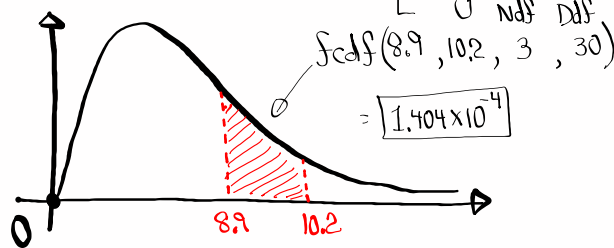
↑  
chi-sqr



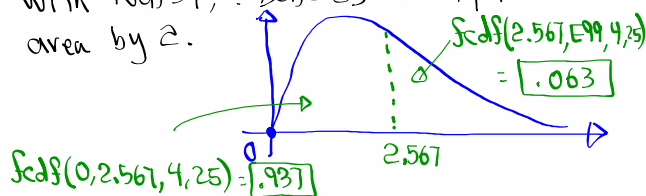
$$2 * \chi^2_{cdf}(0, .025, 8) = \boxed{2.014 \times 10^{-9}}$$

May 2-8:20 AM

Find  $P(8.9 < F < 10.2)$  with  $Ndf=3$  &  $Ddf=30$ .



Find the area on each side of  $F = 2.567$  with  $Ndf=4$ , &  $Ddf=25$ . Multiply the smaller area by 2.



$$2 * \text{Smaller area} = 2 * .063 = \boxed{.126}$$

May 2-8:24 AM